

(Answers calculated by RNC — *caveat emptor*.) In some of these examples I'll quote exact p values, rather than just saying ' $p < 0.05$ '. Don't worry about this — since you're operating from tables and I'm doing some of these questions on a computer to save time, I can quote exact p values when you can't. If I say ' $p = .03$ ', your tables would show that $p < .05$, but not that $p < .01$. If I say ' $p = .125$ ', your tables would show that the answer is not significant at $p = .01$ (i.e. $p > .01$)... and so on.

Q1 glare This question is asking for the 90% confidence intervals. From the formula sheet, we see that we don't know the population mean and SD, but we can work out the sample mean and SD, so we use this formula:

$$\text{Confidence intervals} = \bar{x} \pm \frac{s_X}{\sqrt{n}} t_{critical(n-1)df}$$

(For 95% confidence intervals, use t for $\alpha = 0.05$ two-tailed.)

We want the 90% confidence intervals, though, so we use t for $\alpha = 0.1$ two-tailed. Working as follows:

- sample mean $\bar{x} = 5.015$
- sample standard deviation (SD) = $s_X = 0.711$ (see Examples 2 for a worked example of calculating this by hand, but your calculator should give it to you directly)
- $n = 10$
- standard error of the mean $s_{\bar{x}} = \frac{s_X}{\sqrt{n}} = \frac{0.711}{\sqrt{10}} = 0.225$
- number of degrees of freedom $df = n - 1 = 9$
- with 9 df , critical values of t for 5% each tail = ± 1.833 (if there's 5% in each tail, then 90% of values of t lie within ± 1.833 of the mean). This value is listed in the *Tables and Formulae* sheet as the critical value of t for a two-tailed α of 0.1, or a one-tailed α of 0.05. All these are different ways of saying the same thing!
- the 90% confidence intervals are therefore $5.015 \pm (1.833 \times 0.225) = 5.015 \pm 0.4124 = 4.6026$ and 5.4274

So there's a 90% chance the true mean lies between 4.6026 and 5.4274.

(Do your working with maximum accuracy to avoid rounding errors, but when you've finished it's probably best to express the final answer to 3 significant figures — so we'd state that the 90% confidence intervals are 4.60 and 5.43 to 3 *sf*.)

Q2 nonwd 95% confidence interval: 506–676 ms (to 3 *sf*).
(Same technique as Q1, but with 95% confidence intervals.)
Intermediate steps: $\bar{x} = 590.6667$; $s_X = 133.6259$; $n = 12$; $SEM = 38.57447$; $df = 11$; $t_{critical}$ for $df = 11$ and two-tailed α of 0.05 is 2.201.

Q3 weight (a) 95% confidence interval: 95.3–103.8 g (to 1 *dp*). [Working: $\bar{x} = 99.56$; $s_X = 5.918558$; $n = 10$; $SEM = 1.871612$; $df = 9$; $t_{critical} = 2.262$ for $df = 9$ and two-tailed α of 0.05.]
(b) Since the interval we calculated in (a) includes 100 g, the mean is not significantly different from 100 g at the 5% level. Alternatively, if you wanted to make more work for your good self by practising a one-sample t test in full, you could run a one-sample t test: $t_9 = (\text{mean} - 100 \text{ g}) / SEM = (99.56 - 100) / 1.872 = -0.24$. This is not significant at the 5% level.

Q4 traffic This question calls for an unpaired t test (they're not the same cars each week).

Step 1 – which t test?

Which of the following do we choose from the formula sheet?

- 'Two-sample t test for unrelated samples — where the variances of the two groups are equal'
- 'Two-sample t test for unrelated samples — where the variances of the two groups are unequal'

Well, we can run an F test to decide. We can calculate

$$n_1 = n_2 = 15; \text{ standard deviations } s_1 = 5.53173; s_2 = 5.46243; \text{ variances } s_1^2 = 30.6; s_2^2 = 29.838095$$

To get F , we put the bigger variance on top of the smaller: $F_{n_1-1, n_2-1} = \frac{s_1^2}{s_2^2} = F_{14, 14} = \frac{30.6}{29.838} = 1.0255$. This

is not significant even at the $\alpha = 0.1$ level. So we want the 'equal variances assumed' formula.

Step 2 – the *t* test

Since $n_1 = n_2$, we use this formula (note that $df = 28$):

$$t_{n_1+n_2-2} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = t_{28} = \frac{37.2 - 32.53333}{\sqrt{\frac{30.6}{15} + \frac{29.838}{15}}} = \frac{4.66667}{\sqrt{4.0292}} = 2.325$$

This is significant at $\alpha = 0.05$ two-tailed. Using a computer, we would find that for $t_{28} = 2.325$, we obtain an exact value $p = 0.0276$ (two-tailed).

Comments

- You could argue for the use of a one-tailed *t* test (you'd find $p = 0.0138$ one-tailed) given the question is asking specifically about reductions, but I think a two-tailed test is more sensible — you wouldn't really ignore the result if it turned out that the simulated accident *increased* speeds — and in any case, it doesn't alter the conclusion here: the simulated accident *did* significantly reduce speeds.
- Note that when performing an *F* test for the purpose of deciding which *t* test to use, it is 'conventional' (though not obligatory) to use a decision criterion of $\alpha = 0.05$ for the *F* test (corresponding to the first *F* table in the formula sheet). Since you rigged the *F* ratio so that it's never less than 1, this criterion is equivalent to $\alpha = \mathbf{0.1}$ for the two-tailed question 'are the variances different?' (see Hand-out 3 for explanation).

Q5 cards

No.

This question calls for a paired (two-related-sample) *t* test; in other words using the formula

$$t_{n-1} = \frac{\bar{x} - \mu}{s_{\bar{x}}} = \frac{\bar{x} - \mu}{\frac{s_X}{\sqrt{n}}}$$

First we calculate the *difference* score for each subject (and subsequently ignore the raw scores completely and operate only with the difference scores). For the null hypothesis that there was no effect of the drug, $\mu = 0$. We note that $n = 12$, $df = 11$, mean = 0.833, SEM = 1.359, $t_{11} = (0.833 - 0) / 1.359 = 0.613$; exact two-tailed $p = 0.552$ (if you have access to a computer), NS [not significant].

Q6 digit

No.

Call Arts students group 1 and Science students group 2. We find that

$$n_1 = 11; n_2 = 14$$

standard deviations $s_1 = 0.8311$, $s_2 = 0.7184$

An *F* test (see Q4 for method) shows that the variances are not significantly different; $F_{10,13} = 0.8311^2 / 0.7184^2 = 1.338$ (NS). So we want a two-sample unpaired *t* test assuming equal variances ($df = 23$)... Group sizes are different, so we use this formula:

$$t_{n_1+n_2-2} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}} \text{ where } s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} \text{ (the 'pooled variance estimate')}$$

We calculate

$$s_p^2 = \frac{(11-1) \times (0.8311)^2 + (14-1) \times (0.7184)^2}{11+14-2} = 0.592 \text{ and } t_{23} = \frac{6.745 - 6.807}{\sqrt{\frac{0.592}{11} + \frac{0.592}{14}}} = -0.199, \text{ NS.}$$

Q7 refig

Yes.

Methods just as Q6. Initial *F* test: no significant differences between variances ($F_{9,7} = 1.145$, NS). Two-sample unpaired *t* test assuming equal variances gives $t_{16} = 2.37$, two-tailed $p = 0.03$.

Q8 letters

Yes.

Paired *t* test; $t_9 = -2.299$ (or $+2.299$, depending on which way you calculate the differences), $p = .047$ two-tailed. (Methods just as Q5.)

Q9

Yes.

F test; $F_{8,7} = 9.02$, $p = 0.0089$ two-tailed. (Methods as for the *F* test in Q4.)

Q10

Yes.

F test; $F_{5,4} = 9.98$, $p = .045$ two-tailed. (Methods as for the *F* test in Q4.)