

(Answers calculated by RNC — *caveat emptor*.) In some of these examples I'll quote exact p values, rather than just saying ' $p < 0.05$ '. Don't worry about this — since you're operating from tables and I'm doing some of these questions on a computer to save time, I can quote exact p values when you can't. If I say ' $p = .03$ ', your tables would show that $p < .05$, but not that $p < .01$. If I say ' $p = .125$ ', your tables would show that the answer is not significant at $p = .1$ (i.e. $p > .1$)... and so on.

Q1 Short answer: $U_{4,6} = 5$. Critical value is 3, so not significant (NS).

Step by step:

- Group B is the larger, so group A is 'group 1' and group B is 'group 2'.
- Group A: $n_1 = 4$. Group B: $n_2 = 6$.

Original data:

group 1 (A):	43	39	57	62		
group 2 (B):	51	63	70	55	59	66

Corresponding ranks:

group 1 (A):	2	1	5	7			<i>Sum of ranks</i>
group 2 (B):	3	8	10	4	6	9	15 (= R_1)
							40 (= R_2)

Then $U_1 = R_1 - \frac{n_1(n_1+1)}{2} = 15 - \frac{4 \times 5}{2} = 5$ and $U_2 = R_2 - \frac{n_2(n_2+1)}{2} = 40 - \frac{6 \times 7}{2} = 19$. So U is the smaller of the two, i.e. $U = 5$. We'd write $U_{4,6} = 5$ to indicate n_1 and n_2 as well.

(Just to check our sums: $U_1 + U_2 = 5 + 19 = 24$ and $n_1 n_2 = 4 \times 6 = 24$, so they match, which they must do.

Similarly $R_1 + R_2 = 15 + 40 = 55$ and $\frac{(n_1 + n_2)(n_1 + n_2 + 1)}{2} = \frac{10 \times 11}{2} = 55$ so they also match.)

Now we look up a critical value for $U_{4,6}$ (critical U with $n_1 = 4$ and $n_2 = 6$); we find that it's 3. Since our U is *not* smaller than this, it's *not* significant.

Q2 $U_{7,9} = 15$. Critical value is 13, so NS.

The method is exactly the same as in Q1. Just to make sure you get the ranks right when there are ties, here they are:

Original data:

group 2 (A):	4.5	2.3	7.9	3.4	4.8	2.7	5.6	6.1	3.5
group 1 (B):	3.5	4.9	1.1	2.5	2.3	4.1	0.7		

Corresponding ranks (in bold where ties have been split by taking the mean of the tied ranks):

group 2 (A):	11	3.5	16	7	12	6	14	15	8.5
group 1 (B):	8.5	13	2	5	3.5	10	1		

In this example, no more than two scores are tied for the same rank — but you may come across examples when more scores are tied. The principle is just the same; take the mean of the ranks for which they are tied. So the ranks of {10, 50, 50, 50, 60} are {1, 3, 3, 3, 5}. The ranks of {2.3, 2.3, 2.3, 2.3, 8.1, 8.9} are {2.5, 2.5, 2.5, 2.5, 5, 6}.

Q3 $U_{7,7} = 8$. Critical value is 9, so *significant*.

Q4 $U_{9,10} = 20$. Critical value is 21, so *significant*.

Q5 $U_{16,17} = 76.5$. Critical value is 82, so *significant*.

Q6 $T_7 = 3$. Significant at $\alpha = 0.05$ (one-tailed) or $\alpha = 0.1$ (two-tailed) (critical value 4).

Full working:

Group A	4.5	2.3	7.9	6.8	5.3	6.2	5.7	
Group B	4.3	2.7	9.0	6.7	5.6	10.1	6.9	
Difference (B–A)	–0.2	0.4	1.1	–0.1	0.3	3.9	1.2	
Non-zero differences	(as previous row)							
Ranks of non-zero differences	2	4	5	1	3	7	6	$n = 7$
(ignoring sign)								

Ranks of + differences	4	5	3	7	6	sum = 25 = T ⁺
Ranks of - differences	2		1			sum = 3 = T ⁻

The T statistic is the smaller of T^+ and T^- , i.e. 3. We can write $T_7 = 3$ (to show that $n = 7$). This value, 3, is smaller than the critical value of T_7 for $\alpha = 0.05$ (one-tailed) or $\alpha = 0.1$ (two-tailed), which is 4. But our T is not smaller than the critical value of T_7 for any smaller values of α shown in our tables. So we could say ' $T_7 = 3$, significant at $\alpha = 0.05$ (one-tailed) or $\alpha = 0.1$ (two-tailed)'.

(To check our sums, $T^+ + T^- = 25 + 3 = 28$ and $\frac{n(n+1)}{2} = \frac{7 \times 8}{2} = 28$ so all's well with the world.)

Q7	$T_9 = 3$. Significant at $\alpha = 0.01$ (one-tailed) or $\alpha = 0.02$ (two-tailed) (critical value 4).	
Q8	$T_8 = 8.5$. Not significant ($p > 0.05$ one-tailed; $p > 0.10$ two-tailed; critical value 6).	
Q9	$T_8 = 4$. Significant at $\alpha = 0.05$ (one-tailed) or $\alpha = 0.10$ (two-tailed) (critical value 6).	
	<u>Nonparametric test (subscripts are n, probabilities are two-tailed unless stated):</u>	<u>Parametric equivalent (two-tailed in all cases):</u>
Q10 traffic	Mann-Whitney $U_{15,15} = 59$, $p < .05$ (The question phrases a one-tailed question, but you could argue for a two-tailed test.)	F test for heterogeneity of variance: $F_{14,14} = 1.026$, NS Unpaired t test, equal variances: $t_{28} = 2.325$, $p = .027$
Q11 RT	Wilcoxon matched-pairs signed-rank $T_{12} = 5$, $p < .01$	Paired t test: $t_{11} = 3.879$, $p = .00257$
Q12 cards	Wilcoxon matched-pairs signed-rank $T_{11} = 25$, NS	Paired t test: $t_{11} = 0.613$, NS
Q13 xeno	Mann-Whitney $U_{5,6} = 10$, NS (The question phrases a one-tailed question, but you could argue for a two-tailed test.)	—
Q14 cod	Wilcoxon matched-pairs signed-rank $T_{12} = 11$, $p < .05$	Paired t test: $t_{10} = 2.872$, $p = .0166$
Q15 digits	Mann-Whitney $U_{11,14} = 76.5$, NS	F test for heterogeneity of variance: $F_{10,13} = 1.338$, NS Unpaired t test, equal variances: $t_{23} = 0.199$, NS
Q16 revfig	Mann-Whitney $U_{8,10} = 16$, $p < .05$ (The question phrases a one-tailed question, but you could argue for a two-tailed test.)	F test for heterogeneity of variance: $F_{9,7} = 1.146$, NS Unpaired t test, equal variances: $t_{16} = 2.278$, $p = .031$
Q17 conv	Wilcoxon matched-pairs signed-rank $T_{12} = 13.5$, $p < .05$	Paired t test: $t_{11} = 2.218$, $p = .0485$
Q18 bats	Wilcoxon matched-pairs signed-rank $T_9 = 3.5$, $p < .02$	Paired t test: $t_9 = 2.743$, $p = .0228$
Q19 music	Mann-Whitney $U_{10,10} = 48$, NS	F test for heterogeneity of variance: $F_{9,9} = 1.327$, NS Unpaired t test, equal variances: $t_{18} = 0.051$, NS
Q20 letters	Wilcoxon matched-pairs signed-rank $T_9 = 5.5$, $p < .05$	Paired t test: $t_9 = 2.299$, $p = .0471$
Q21 vote	Mann-Whitney $U_{8,8} = 4$, $p < .05$	—
Q22 rats	Mann-Whitney $U_{10,10} = 40$, NS	F test for heterogeneity of variance: $F_{9,9} = 1.899$, NS Unpaired t test, equal variances: $t_{18} = 0.051$, NS
Q23 radar	Wilcoxon matched-pairs signed-rank $T_{12} = 12$, $p < .05$	Paired t test: $t_{11} = 2.449$, $p = .0323$
Q24 col'r	Wilcoxon signed-rank $T_{16} = 37$, NS (The question phrases a one-tailed question, but you could argue for a two-tailed test.)	One-sample t test: $t_{15} = 1.730$, NS.

Q25

Use the normal approximation for U . If $U_{20,60} = 400$, then $z = \frac{U - \frac{n_1 n_2}{2}}{\sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}} = \frac{400 - 600}{\sqrt{\frac{1200 \times 81}{12}}} = -2.22$

This Z score is associated with a p value of 0.0132 (one-tailed) or $2 \times 0.0132 = 0.0264$ (two-tailed).