Split-split Plot Arrangement

The split-split plot arrangement is especially suited for three or more factor experiments where different levels of precision are required for the factors evaluated.

This arrangement is characterized by:

1. Three plot sizes corresponding to the three factors; namely, the largest plot for the main factor, the intermediate size plot for the subplot factor, and the smallest plot for the sub-subplot factor.

2. There are three levels of precision with the main plot factor receiving the lowest precision, and the sub-subplot factor receiving the highest precision.

Example

Split-split plot arrangement randomized as an RCBD. Three levels of the whole plot factor, A, two levels of the subplot factor, B, and three levels of the sub-subplot factor, C. Diagram shows the first replicate.

<table>
<tr><td rowspan="2">a_0 Whole plot</td><td>a_1b_0 subplots</td><td>$a_2b_1c_0$</td><td>$a_2b_1c_2$</td><td>$a_2b_1c_1$</td></tr>
<tr><td>a_1b_1 subplots</td><td>$a_2b_0c_1$</td><td>$a_2b_0c_0$</td><td>$a_2b_0c_2$</td></tr>
</table>

Randomization Procedure

The randomization procedure for the split-split plot arrangement consists of three parts:

1. Randomly assign whole plot treatments to whole plots based on the experimental design used.
2. Randomly assign subplot treatments to the subplots.
3. Randomly assign sub-subplot treatments to the sub-subplots.

The experimental design used to randomize the whole plots will not affect randomization of the sub and sub-subplots.

Expected Mean Squares for the Split-split Plot Arrangement

The example to be given will be for an RCBD with factor A as the whole plot factor, factor B as the subplot factor, and factor C as the sup-subplot factor. Factors A, B, and C will be considered random effects.

Source of variation	df	Expected mean square
Replicate	r-1	$\sigma^2 + c\sigma^2_\theta + bc\sigma^2_\gamma + abcr\sigma^2_R$
A	a-1	$\sigma^2 + c\sigma^2_\theta + bc\sigma^2_\gamma + r\sigma^2_{ABC} + rb\sigma^2_{AC} + rc\sigma^2_{AB} + rbc\sigma^2_A$
Error (a) = RepxA	(r-1)(a-1)	$\sigma^2 + c\sigma^2_\theta + bc\sigma^2_\gamma$
B	b-1	$\sigma^2 + c\sigma^2_\theta + r\sigma^2_{ABC} + ra\sigma^2_{BC} + rc\sigma^2_{AB} + rac\sigma^2_B$
AxB	(a-1)(b-1)	$\sigma^2 + c\sigma^2_\theta + r\sigma^2_{ABC} + rc\sigma^2_{AB}$
Error (b) = RepxB(A)	a(r-1)(b-1)	$\sigma^2 + c\sigma^2_\theta$
C	c-1	$\sigma^2 + r\sigma^2_{ABC} + ra\sigma^2_{BC} + rb\sigma^2_{AC} + rab\sigma^2_C$
AxC	(a-1)(c-1)	$\sigma^2 + r\sigma^2_{ABC} + rb\sigma^2_{AC}$
BxC	(b-1)(c-1)	$\sigma^2 + r\sigma^2_{ABC} + ra\sigma^2_{BC}$
AxBxC	(a-1)(b-1)(c-1)	$\sigma^2 + r\sigma^2_{ABC}$
Error (c) = RepxC(AxB)	ab(r-1)(c-1)	σ^2
Total	rabc-1	

ANOVA of a Split-split Plot Arrangement - Table 1. Data for split-split plot example

Treatments			Replicates				Treatment totals	
A_j	B_k	C_l	1	2	3	4		
0	0	0	25.7	25.4	23.8	22.0	96.9	
0	0	1	31.8	29.5	28.7	26.4	116.4	
0	0	2	34.6	37.2	29.1	23.7	124.6	
Subplot tot. $Y_{i00.}$			92.1	92.1	81.6	72.1	337.9=	$Y_{.00.}$
0	1	0	27.7	30.3	30.2	33.2	121.4	
0	1	1	38.0	40.6	34.6	31.0	144.2	
0	1	2	42.1	43.6	44.6	42.7	173.0	
Subplot tot. $Y_{i01.}$			107.8	114.5	109.4	106.9	438.6=	$Y_{.01.}$
Whole plot tot. $Y_{i0..}$			199.9	206.6	191.0	179.0	776.5=	$Y_{.0..}$
1	0	0	28.9	24.7	27.8	23.4	104.8	
1	0	1	37.5	31.5	31.0	27.8	127.8	
1	0	2	38.4	32.5	31.2	29.8	131.9	
Subplot tot. $Y_{i10.}$			104.8	88.7	90.0	81.0	364.5=	$Y_{.10.}$
1	1	0	38.0	31.0	29.5	30.7	129.2	
1	1	1	36.9	31.9	31.5	35.9	136.2	
1	1	2	44.2	41.6	38.9	37.6	162.3	
Subplot tot. $Y_{i11.}$			119.1	104.5	99.9	104.2	427.7=	$Y_{.11.}$
Whole plot tot. $Y_{i1..}$			223.9	193.2	189.9	185.2	792.2=	$Y_{.1..}$
2	0	0	23.4	24.2	21.2	20.9	89.7	
2	0	1	25.3	27.7	23.7	24.3	101.0	
2	0	2	29.8	29.9	24.3	23.8	107.8	
Subplot tot. $Y_{i20.}$			78.5	81.8	69.2	69.0	298.5=	$Y_{.20.}$
2	1	0	20.8	23.0	25.2	23.1	92.1	
2	1	1	29.0	32.0	26.5	31.2	118.7	
2	1	2	36.6	37.8	34.8	40.2	149.4	
Subplot tot. $Y_{i21.}$			86.4	92.8	86.5	94.5	360.2=	$Y_{.21.}$
Whole plot tot. $Y_{i2..}$			164.9	174.6	155.7	163.5	658.7=	$Y_{.2..}$
Rep total $Y_{i...}$			588.7	574.4	536.6	527.7	2227.4=	$Y_{....}$

Table 2. Totals for two-way interactions.

	A X B ($Y_{.jk.}$)		A X C ($Y_{.j.l}$)				B X C ($Y_{..kl}$)	
	b_0	b_1	c_0	c_1	c_2		b_0	b_1
a_0	337.9	438.6	218.3	260.6	297.6	c_0	291.4	342.7
a_1	364.5	427.7	234.0	264.0	294.2	c_1	345.2	399.1
a_2	298.5	360.2	181.8	219.7	257.2	c_2	364.3	484.7

Table 3. Totals for main effects.

A ($Y_{.j..}$)			B ($Y_{..k.}$)		C ($Y_{...l}$)		
a_0	a_1	a_2	b_0	b_1	c_0	c_1	c_2
776.5	792.2	658.7	1000.9	1226.5	634.1	744.3	849.0

Step 1. Calculate correction factor

$$CF = \frac{Y_{....}^2}{rabc}$$

$$= \frac{(2227.4)^2}{(4x3x2x3)}$$

$$= 68,907.094$$

Step 2. Calculate total sum of squares

$$\text{Total SS} = \sum Y_{ijkl}^2 - CF$$

$$\text{Total SS} = (25.7^2+25.4^2+23.8^2+\cdots+40.2^2) - \text{CF} = 2840.606$$

Step 3. Calculate replicate sum of squares

$$\text{Rep SS} = \frac{\sum Y_{i...}^2}{abc} - CF$$

$$= \frac{(588.7^2 + 574.4^2 + 536.6^2 + 527.7^2)}{3x2x3} - CF$$

$$= 143.456$$

Step 4. Calculate A sum of squares.

$$\text{A SS} = \frac{\sum Y_{.j..}^2}{rbc} - CF$$

$$=$$

$$= \frac{(776.5^2 + 792.2^2 + 658.7^2)}{4x2x3} - CF$$

$$= 443.689$$

Step 5. Calculate Whole plot sum of squares.

$$\text{Whole Plot SS} = \frac{\sum Y_{ij}^2}{bc} - CF$$

$$= \frac{(199.9^2 + 206.6^2 + 191.0^2 + ... + 163.5^2)}{bc} - CF$$

$$= 698.903$$

Step 6. Calculate Error(a) sum of squares.

Error (a) SS = Whole plot SS - A SS - Rep SS

= 698.903 - 443.689- 143.456 = 111.749

Step 7. Calculate B sum of squares.

$$\text{B SS} = \frac{\sum Y_{..k.}^2}{rac} - CF$$

$$= \frac{(1000.9^2 + 1226.5^2)}{4x3x3} - CF$$

$$= 706.880$$

Step 8. Calculate A X B sum of squares.

$$\text{AxB SS} = \frac{\sum Y_{.jk.}^2}{rc} - CF - \text{A SS - B SS}$$

$$= \frac{(337.9^2 + 364.5^2 + 298.5^2 + ... + 438.6^2)}{4x3} \text{ - CF - A SS - B SS}$$

$$= 40.687$$

Step 9. Calculate subplot sum of squares.

$$\text{Subplot SS} = \frac{\sum Y_{ijk.}^2}{c} - CF$$

$$= (92.1^2 + 92.1^2 + 81.6^2 + ... + 94.5^2) \big/ 3 - CF$$

$$= 1524.813$$

Step 10. Calculate Error(b) sum of squares.

Error(b) SS = Subplot SS - A X B SS - B SS - Error(a) SS - A SS - Rep SS

= 1524.813 - 40.687 - 706.88 - 111.749 - 443.689 - 143.465 = 78.343

Step 11. Calculate C sum of squares.

$$C\,SS = \frac{\sum Y_{...l}^2}{rab} - CF$$

$$= \frac{(634.1^2 + 744.3^2 + 849.0^2)}{4x3x2} - CF$$

$$= 962.335$$

Step 12. Calculate A X C sum of squares.

$$AxC\,SS = \frac{\sum Y_{.j.l}^2}{rb} - CF\text{ - }A\,SS\text{ - }C\,SS$$

$$= \frac{(218.3^2 + 234.0^2 + 181.3^2 + ... + 257.2^2)}{4x2}\text{ - }CF\text{ - }A\,SS\text{ - }C\,SS$$

$$= 13.1097$$

Step 13. Calculate B X C sum of squares.

$$BxC\,SS = \frac{\sum Y_{..kl}^2}{ra} - CF\text{ - }B\,SS\text{ - }C\,SS$$

$$= \frac{(291.4^2 + 345.2^2 + 364.3^2 + ... + 484.7^2)}{4x3}\text{ - }CF\text{ - }B\,SS\text{ - }C\,SS$$

$$= 127.831$$

Step 14. Calculate AxBxC sum of squares.

$$\text{AxBxC SS} = \frac{\sum Y^2_{.jkl}}{r} \text{ - CF - A SS - B SS - C SS - AxB SS - AxC SS - BxC SS -}$$

$$= \frac{(96.9^2 + 116.4^2 + 124.6^2 + ... + 149.4^2)}{4} - \text{CF - A SS - B SS - C SS - AxB SS - AxC SS - BxC SS}$$

$$= 44.019$$

Step 15. Calculate Error(c) sum of squares.

Error(c) SS = Total SS-AxBxC SS-BxC SS-AxC SS-C SS-Error(b) SS- AxB SS-B SS-Error(a) SS-A SS-Rep SS

= 168.498

Step 16 ANOVA

SOV	df	SS	MS	F (A,B, C fixed)
Replicate	3	143.45	47.819	
A	2	443.689	221.844	A MS/Error(a) MS = 11.91**
Error(a)	6	111.749	18.626	
B	1	706.88	706.88	B MS/Error(b) MS = 81.21**
A X B	2	40.688	20.344	AxB MS/Error(b) MS = 2.34
Error(b)	9	78.343	8.705	
C	2	962.335	481.168	C MS/Error(c) MS = 102.80**
A X C	4	13.110	3.277	AxC MS/Error(c) MS = 0.70
B X C	2	127.831	63.915	BxC MS/Error(c) MS = 13.66**
A X B X C	4	44.019	11.005	AxBxC MS/Error(c) MS = 2.35
Error(c)	36	168.498	4.681	
Total	71	2840.606		

LSD's for Split-split Plot Arrangement

1. To compare two whole plot means averaged over all sub and sub-sub-plot treatments (e.g. a_0 vs. a_1).

$$LSD = t_{\frac{\alpha}{2},Err(a)df}\sqrt{\frac{2Error(a)MS}{rbc}}$$

$$= 2.447\sqrt{\frac{2(18.626)}{4x2x3}}$$

$$= 3.05$$

2. To compare two subplot means averaged over all whole and sub-sub plot treatments (e.g. b_0 vs. b_1).

$$LSD = t_{\frac{\alpha}{2},Err(b)df}\sqrt{\frac{2Error(b)MS}{rac}}$$

$$= 2.262\sqrt{\frac{2(8.705)}{4x3x3}}$$

$$= 1.57$$

3. To compare two sub-subplot means averaged over all whole and subplot treatments (e.g. c_0 vs c_1).

$$LSD = t_{\frac{\alpha}{2},Err(c)df}\sqrt{\frac{2Error(c)MS}{rab}}$$

$$= 2.030\sqrt{\frac{2(4.681)}{4x3x3}2}$$

$$= 1.27$$

4. To compare two subplot means (averaged over all sub-subplot treatments) at the same levels of the whole plot (e.g. a_0b_0 vs. a_ob_1).

$$\mathrm{LSD} = t_{\frac{\alpha}{2},\mathrm{Err(b)df}} \sqrt{\frac{2\mathrm{Error(b)MS}}{\mathrm{rc}}}$$

$$= 2.262\sqrt{\frac{2(8.705)}{4\mathrm{x}3}}$$

$$= 2.72$$

5. To compare two whole plot means (averaged over all sub-subplot treatments) at the same or different levels of the subplot (e.g. a_0b_0 vs a_1b_0 or a_0b_0 vs a_2b_1).

$$\mathbf{LSD} = \mathbf{t}_{\mathbf{AB}} \sqrt{\frac{\mathbf{2[(b-1)Error(b)MS + Error(a)\,MS]}}{\mathbf{rbc}}}$$

and

$$\mathbf{t}_{\mathbf{AB}} = \frac{\mathbf{(b-1)Error(b)MS}\left(\mathbf{t}_{\frac{\alpha}{2},\mathbf{Err(b)df}}\right) + \mathbf{Error(a)MS}\left(\mathbf{t}_{\frac{\alpha}{2},\mathbf{Err(a)df}}\right)}{\mathbf{(b-1)Error(b)\,MS + Error(a)\,MS}}$$

$$\therefore$$

$$\mathbf{t}_{\mathbf{AB}} = \frac{\mathbf{(2-1)(8.705)(2.262) + 18.626(2.447)}}{\mathbf{(2-1)8.705 + 18.626}}$$

$$= \mathbf{2.388}$$

and

$$\mathbf{LSD} = \mathbf{2.}388\sqrt{\frac{\mathbf{2[(2-1)8.705 + 18.626]}}{\mathbf{4x2x3}}}$$

6. To compare two sub-subplot means (averaged over all subplot treatments) at the same levels of the whole plot (e.g. a_0c_0 vs. a_oc_1).

$$LSD = t_{\frac{\alpha}{2},Err(c)df}\sqrt{\frac{2Error(c)MS}{rb}}$$

$$= 2.030\sqrt{\frac{2(4.681)}{4x2}}$$

$$= 2.20$$

7. To compare two whole plot means (averaged over all subplot treatments) at the same or different levels of the sub-subplot (e.g. a_0c_0 vs. a_1c_0 or a_0c_0 vs. a_2c_1).

$$LSD = t_{AC}\sqrt{\frac{2[(c-1)Error(c)MS + Error(a)\,MS]}{rbc}}$$

and

$$t_{AC} = \frac{(c-1)Error(c)MS\left(t_{\frac{\alpha}{2},Err(c)df}\right) + Error(a)MS\left(t_{\frac{\alpha}{2},Err(a)df}\right)}{(c-1)Error(c)\,MS + Error(a)\,MS}$$

$$\therefore$$

$$t_{AC} = \frac{(3-1)(4.681)(2.030) + 18.626(2.447)}{(3-1)4.681 + 18.626}$$

$$= 2.307$$

and

$$LSD = 2.307\sqrt{\frac{2[(3-1)4.681 + 18.626]}{4x2x3}}$$

$$= 3.52$$

8. To compare two sub-subplot means (averaged over all whole plot treatments) at the same levels of the subplot (e.g. b_0c_0 vs. b_oc_1).

$$LSD = t_{\frac{\alpha}{2},Err(c)df}\sqrt{\frac{2Error(c)MS}{ra}}$$

$$= 2.030\sqrt{\frac{2(4.681)}{4x3}}$$

$$= 1.79$$

9. To compare two subplot means (averaged over all whole plot treatments) at the same or different levels of the sub-subplot (e.g. b_0c_0 vs. b_1c_0 or b_0c_0 vs. b_2c_1).

$$LSD = t_{BC}\sqrt{\frac{2[(c-1)Error(c)MS + Error(b)\,MS]}{rac}}$$

and

$$t_{BC} = \frac{(c-1)Error(c)MS\left(t_{\frac{\alpha}{2},Err(c)df}\right) + Error(b)MS\left(t_{\frac{\alpha}{2},Err(b)df}\right)}{(c-1)Error(c)\,MS + Error(b)\,MS}$$

$\therefore$

$$t_{BC} = \frac{(3-1)(4.681)(2.030) + 8.705(2.262)}{(3-1)4.681 + 8.705}$$

$$= 2.142$$

and

$$LSD = 2.142\sqrt{\frac{2[(3-1)4.681 + 8.705]}{4x3x3}}$$

$$= 2.15$$

10. To compare two sub-subplot means at the same combination of whole plot and subplot treatments (e.g. $a_0b_0c_0$ vs. $a_0b_0c_2$).

$$LSD = t_{\frac{\alpha}{2},Err(c)df}\sqrt{\frac{2Error(c)MS}{r}}$$

$$= 2.030\sqrt{\frac{2(4.681)}{4}}$$

$$= 3.11$$

11. To compare two subplot means at the same level of whole plot and sub-subplot (e.g. $a_0b_0c_0$ vs. $a_0b_1c_0$).

$$LSD = t_{ABC}\sqrt{\frac{2[(c-1)Error(c)MS + Error(b)\,MS]}{rc}}$$

and

$$t_{ABC} = \frac{(c-1)Error(c)MS\left(t_{\frac{\alpha}{2},Err(c)df}\right) + Error(b)MS\left(t_{\frac{\alpha}{2},Err(b)df}\right)}{(c-1)Error(c)\,MS + Error(b)\,MS}$$

$\therefore$

$$t_{ABC} = \frac{(3-1)(4.681)(2.030) + 8.705(2.262)}{(3-1)4.681 + 8.705}$$

$$= 2.142$$

and

$$LSD = 2.142\sqrt{\frac{2[(3-1)4.681 + 8.705]}{4x3}}$$

$$= 3.72$$

12 To compare two whole plot means at the same combination of subplot and sub-subplot treatments ($a_0b_0c_0$ vs. $a_1b_0c_0$).

$$\mathbf{LSD} = \mathbf{t}_{\mathbf{ABC}}\sqrt{\frac{\mathbf{2[(b)(c-1)Error(c)MS + (b-1)Error(b)\,MS + Error(a)MS]}}{\mathbf{rbc}}}$$

and

$$\mathbf{t}_{\mathbf{ABC}} = \frac{\mathbf{(b)(c-1)Error(c)MS}\left(\mathbf{t}_{\frac{\alpha}{2},\mathbf{Err(c)df}}\right) + \mathbf{(b-1)Error(b)MS}\left(\mathbf{t}_{\frac{\alpha}{2},\mathbf{Err(b)df}}\right) + \mathbf{Error(a)MS}\left(\mathbf{t}_{\frac{\alpha}{2},\mathbf{Err(a)df}}\right)}{\mathbf{(b)(c-1)Error(c)\,MS + (b-1)Error(b)\,MS + Error(a)MS}}$$

$\therefore$

$$\mathbf{t}_{\mathbf{ABC}} = \frac{\mathbf{((2*(3-1)(4.681)(2.030)} + (2-1)\mathbf{8.}705(\mathbf{2.262}) + 18.626(2.447)\mathbf{)}}{\mathbf{(2*(3-1)4.681)} + (2-1)\mathbf{8.}705 + 18.626}$$

$= \mathbf{2.242}$

and

$$\mathbf{LSD} = 2.242\sqrt{\frac{\mathbf{2[(2*(3-1)4.681)} + (2-1)\mathbf{8.}705 + 18.626\mathbf{]}}{\mathbf{4x2x3}}}$$

$= 4.39$